TQIC Kingscote

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THE SLIDE RULE

(PREVIOUSLY MG 114)

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LIBRARY ISSUE (NOT TO BE REMOVED)

1. INTRODUCTION.

1.1 The slide rule is a device designed to reduce the time taken to work out arithmetical calculations. When the principles of operation of the slide rule have been mastered, the speed at which routine calculations can be carried out makes its advantages apparent both in the solution of everyday problems and in examinations.

1.2 As you work through the exercises in this book, cover the answers given until you have a solution, then check immediately to verify the accuracy of your working.

Issue 2, 1968.

2. DESCRIPTION.

2.1 Fig. 1 shows the face of the slide rule we will discuss.

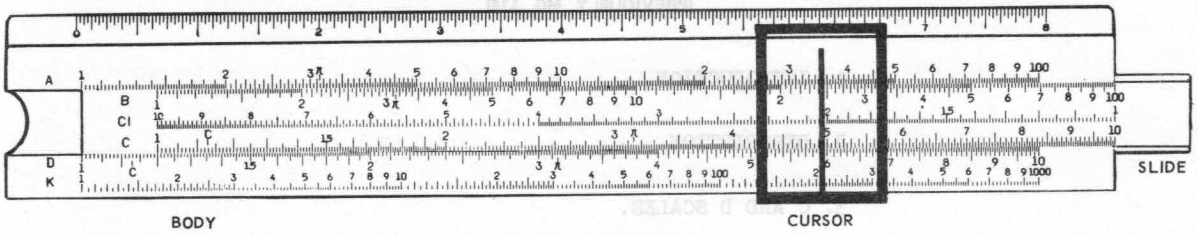

FIG. 1. A SLIDE RULE.

The "body" of the rule carries the A, D and K scales, associated with squaring, multiplication and cubing, respectively. The "slide" carries the B, C, and CI scales and can be moved to "slide" its scales past the fixed scales on the "body". B and C scales have the same divisions as A and D respectively, while the Inverse C (CI) is the reciprocal of the C scale.

The "cursor" can be moved along the length of the rule, and will stay in the position in which it is set. The "hairline" engraved on the cursor can then be used to locate a scale reading for future reference.

2.2 Scale Divisions - Each scale is divided in such a way that the distance of any number on a scale from the beginning of that scale is related to the logarithm of that number. That is, although the scale is marked 1, 2, 3..... the distances 1 to 2, 1 to 3, and so on are proportional to the logarithms of the numbers.

The logarithm of 1 is 0, therefore 1 appears at the zero mark, or beginning, of each scale.

Exercise. Divide a page into three columns
Head the columns as shown below:-

Number	log	log × 5(inches)
In the left column write the numbers 1, 1.5, 2, 3, to 10.	In the centre column write the logs of these numbers, 0, 0.18, 0.30, to 1.	In the third column write the result of these log values multiplied by 5.

On a line 5 inches long, mark off the distances shown in the third column, but designate each mark by the associated number in the left column. The resultant scale should look very much like the C and D scales of the rule illustrated in Fig. 1.

2.3 Accuracy - The accuracy of any result obtained by manipulation of a slide rule is limited by the scale divisions provided, which are limited by the length of the scale. However, the results obtained will generally be within the accuracy of tables of logarithms, and much more accurate than the value given to electrical circuit components. In general, three significant figures will give sufficient accuracy for any component of a communications circuit, so slide rule accuracy is more than sufficient for all practical needs.

2.4 Approximation - When there are more than three significant figures in a number to be set on a slide rule, the process of "rounding off" can be applied without greatly affecting accuracy. A number is "rounded off" by dropping one or more figures at its right, and replacing the figure dropped by a zero.

If the last figure dropped is 6 or more, increase the last figure retained by one. 4978 would be rounded off to 4980. If the last figure dropped is 4 or less, leave the last figure retained as it is. 4243 would be rounded off to 4240 or 4200.

If the last figure dropped is 5, the last figure retained, if not even, should have 1 added. 55.75 would become 55.8 but 68.85 would become 68.8.

3. C AND D SCALES.

3.1 These identical scales are used for multiplication and division. (Fig. 2).

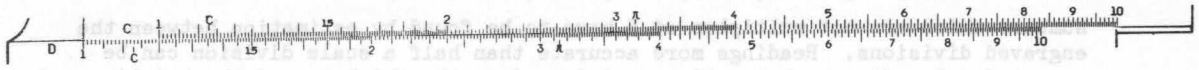

FIG. 2. C AND D SCALES OF A SLIDE RULE.

The scale is divided into nine main units covering the numbers 1 to 10. Each of these units is divided into smaller divisions. The scales are read as a normal rule is read, the only differences being that the main units become smaller as 10 is approached, and the divisions within a unit depend upon the space available.

3.2 Consider unit 1 (that from 1 to 2). There are ten major divisions each equal to 0.1 of the unit. Each major division is then subdivided into ten divisions, each representing 1/10th of the major divisions, or 1/100th (0.01) of the unit 1 to 2.

The major divisions from 1 to 2 are then read as 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8 and 1.9. (Fig. 3).

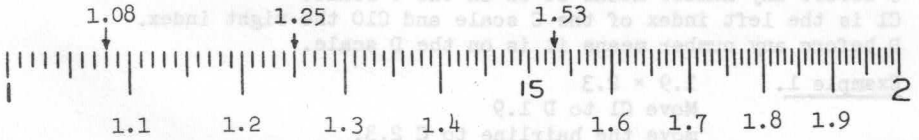

FIG. 3. C AND D SCALE 1 TO 2.

Three significant figures can be read directly from the rule e.g. 1.08, 1.25, 1.53.

3.3 The sections 2 to 3 and 3 to 4 are again divided into ten major divisions each equal to 0.1 of the unit. Each major division is then subdivided into five divisions each equal to 0.2 of the major division, or 0.02 of the section 2 to 3 or 3 to 4.

The major divisions from 2 to 4 are then read as shown in Fig. 4.

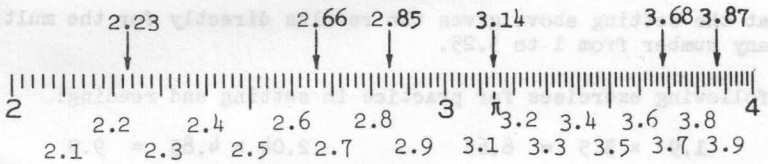

FIG. 4. C AND D SCALE 2 TO 4.

Even numbers of three significant figures can be read directly from the scale, 2.66, 3.14, 3.68. Odd numbers of three significant figures can be read by estimation or "splitting" divisions, e.g. 3.87, 2.85, 2.23.

3.4 The sections 4 to 10 are also divided into ten major divisions each equal to 0.1 of the main unit. Each major division is then subdivided once only, so the smallest divisions have a value of one-half of a major division or 0.05 per division.

The major divisions from 4 to 10 are read as shown in Fig. 5.

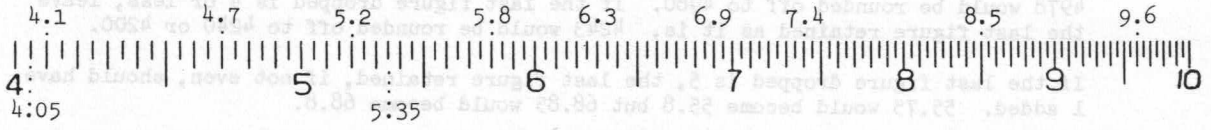

FIG. 5. C AND D SCALE 4 TO 10.

Numbers which are multiples of 5 can be read to three significant figures directly from the scale, e.g. 4.05, 5.35, 6.25, 7.15, 8.75, 9.95.

Numbers which are not multiples of 5 have to be found by estimation between the engraved divisions. Readings more accurate than half a scale division can be expected only after much practice, and depend on the thickness of the hairline and the length of the rule.

4. MULTIPLICATION.

4.1 The C and D scales are used for multiplication as these scales provide the best accuracy. Using logarithms, one number can be multiplied by another by the process of adding the logarithms of the numbers, and finding the antilog of that sum to obtain the answer. As the slide rule scales are divided logarithmically, multiplication can be performed by adding the scale lengths representing the numbers.

The answer can be read directly, as the scale is graduated in the numbers the lengths represent.

4.2 The following abbreviations will now apply;

C before any number means it is on the C scale.
C1 is the left index of the C scale and C10 the right index.
D before any number means it is on the D scale.

Example 1. 1.9 × 2.3
Move C1 to D 1.9
move the hairline to C 2.3.
Read the answer on the D scale: 4.37.

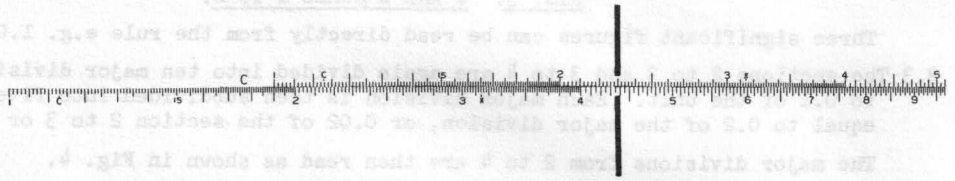

FIG. 6. RULE SET TO EXAMPLE 1.

Note that the setting above gives the results directly for the multiplication of 1.9 by any number from 1 to 5.25.

4.3 Try the following exercises for practice in setting and reading:

1.9 × 3.5 = 6.65	2.04 × 4.85 = 9.9
1.5 × 2.2 = 3.3	3.36 × 2.54 = 8.54
1.67 × π = 5.25	4.05 × 1.84 = 7.45

If your results are not within the thickness of the cursor hairline of these results, you are making poor slide settings.

Example 2. 2.42×4.75.
Moving C1 to D 2.42 puts C 4.75 off the scale.

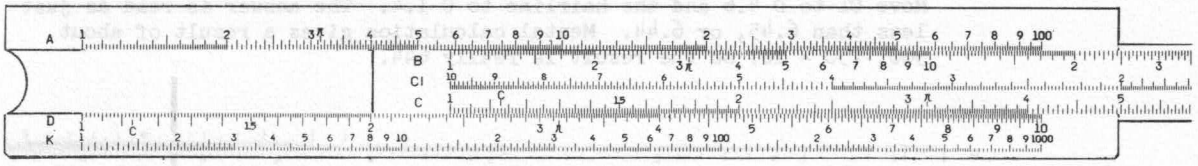

FIG. 7. EXAMPLE 2.

This shows that the answer lies on an imaginary D scale to the right, which would cover the numbers 10 to 100.

To solve this problem, move the slide so that C10 is set to D 2.42 and move the hairline to C 4.75 to read the answer at D 1.15. Our D scale has now taken the place of the imaginary scale to the right, which covers the numbers 10 to 100. In other words, every number on the D scale now represents the number 10 times its face value. Therefore our result should be read as 1.15×10, or 11.5.

A quick mental calculation would also locate our decimal point correctly. Rounding off the numbers (para. 2.4) 2.42 and 4.75 would give 2×5, so the answer would be about 10. The scale reading 1.15 must then represent 11.5, not 0.115, 1.15 or 115.

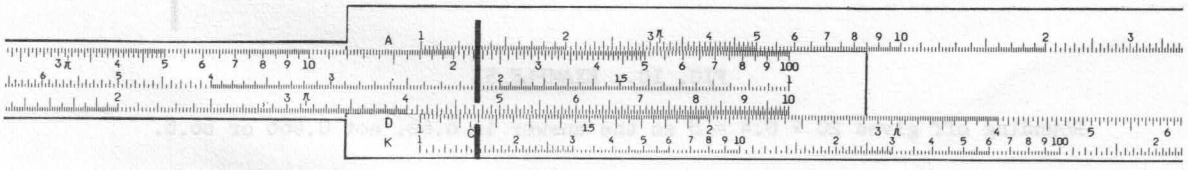

FIG. 8. EXAMPLE 2.

Note that the setting gives the results directly for the multiplication of 2.42 by any number from 4.135 to 10.

Example 3. Reading RMS values and Peak values:

RMS = 0.707 Peak.
Set C1 or C10 to 7.07 on the D scale.

All values on the C scale are now multiplied by 0.707 and Peak values read on the C scale have their RMS equivalents directly opposite on the D scale.

Conversely, with this same setting, RMS values on the D scale can be converted to their Peak equivalents by reading directly to the C scale.

Exercise. Complete the following Table.

Peak volts	339	20			52	1.23	
R.M.S. volts	240		20	6.3			0.905

4.4 Try the following exercises for practice in setting and locating the decimal point.

	Mentally	Slide Rule	Arithmetically
14.14×7	98	$9.9 \rightarrow 99$	98.98
28.25×5.25	150	$1.485 \rightarrow 148.5$	148.3125
8.9×6.8	63	$6.05 \rightarrow 60.5$	60.52
36.8×3.86	148	$1.42 \rightarrow 142$	142.048
0.87×4.65	4.5	4.05	4.0455
1.28×2.08	2	2.66	2.6624

Example 4. 46 × 14

Move C1 to D 4.6 and the hairline to C 1.4. The answer is read as just less than 6.45, or 6.44. Mental calculation gives a result of about 500, (50 × 10) so the result is really 644.

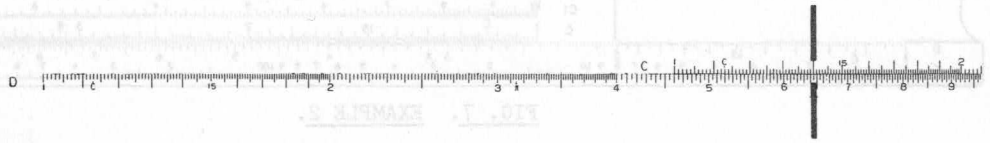

FIG. 9. EXAMPLE 4.

Example 5. 18.72 × 0.356.

Move C1 to D 1.872 (about the thickness of the engraved index past 1.87) and the hairline to C 3.56. The answer appears on the D scale just to the right of 6.65 so would be about 6.66 by estimation.

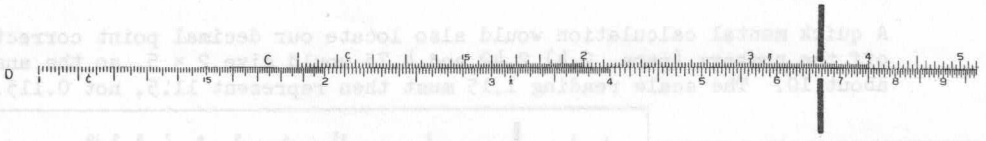

FIG. 10. EXAMPLE 5.

Rounding off gives 20 × 0.4 = 8 so the answer is 6.66, not 0.666 or 66.6.

4.5 Work through the following exercises, to find the power in watts;

Amps	Volts		Mentally	Slide Rule	Arithmetically
10.9	× 26.8	= 1.09 × 2.68 × 10^2	10 × 27 = 270	2.92 → 292	293.02
14.45	× 31.8	= 1.445 × 3.18 × 10^2	10 × 30 = 300	4.6 → 460	459.510
21.5	× 32.4	= 2.15 × 3.24 × 10^2	20 × 30 = 600	6.97 → 697	696.60
0.355	× 0.182	= 3.55 × 1.82 × 10^{-2}	0.4 × 0.2= 0.08	6.46 → 0.0646	0.06461
0.4375	× 21.1	= 4.375 × 2.11	0.4 × 20 = 8	9.24	9.23125

4.6 Now try the following exercises which will involve "end-switching", using C10 instead of C1.

Ohms	Amps		Mentally	Slide Rule	Arithmetically
34	× 7.2	= 3.4 × 7.2 × 10	30 × 7 = 210	2.45 → 245	244.8
408	× 0.0036	= 4.08 × 3.6 × 10^{-1}	$\frac{400 \times 4}{1000}$ = 1.6	1.47	1.4688
75.2	× 485	= 7.52 × 4.85 × 10^3	80 × 500= 40,000	36.5 → 36,500	36,472
0.807	× 0.274	= 8.07 × 2.74 × 10^{-2}	0.8 × 0.3= 0.24	2.21 → 0.221	0.221118
90.3	× 164	= 9.03 × 1.64 × 10^3	90 × 200= 18,000	1.48 → 14,800	14,809.2

Example 6. $1.84 \times 3.6 \times 2.02$

This type of problem involves two slide settings.
Set C1 to D 1.84.
Move the cursor hairline to C 3.6.

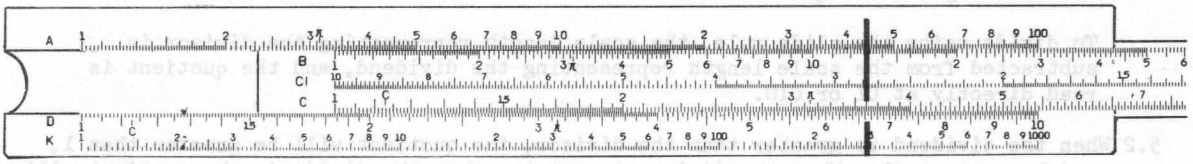

FIG. 11. EXAMPLE 6 PART 1.

Move the slide to bring C10 to the hairline
Move the cursor hairline to C 2.02 to read D 1.34.

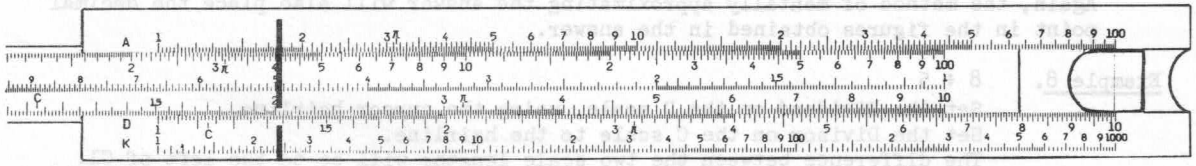

FIG. 12. EXAMPLE 6 PART 2.

Mental calculation: $2 \times 4 \times 2 = 16$ so the answer is 13.4.

4.7 The following exercises will combine all the skills learned to this point. Remember to use a mental approximation to locate the decimal point. You should now be convinced of the accuracy of the slide rule result, so the arithmetical result will not be shown.

	Mentally	Slide Rule
$15.6 \times \pi \times 0.8$	$(15 \times 3 \times 1) = 45$	$3.92 \rightarrow 39.2$
$191 \times 33.4 \times 0.163$	$(200 \times 30 \times 0.2) = 1200$	$1.04 \rightarrow 1040$
$1.6 \times 45.5 \times 144$	$(2 \times 50 \times 100) = 10,000$	$1.05 \rightarrow 10,500$
$0.216 \times 17.1 \times 305$	$(0.2 \times 20 \times 300) = 1200$	$1.13 \rightarrow 1130$
$63.5 \times 21.4 \times 0.0815$	$(60 \times 20 \times 0.1) = 120$	$1.11 \rightarrow 111$

Example 7. Calculate the reactance of a coil with an inductance of 148mH connected to a 1200Hz supply.

Solution: $X_L = 2\pi fL$
$= 2\pi \times 1200 \times 0.148$

Set C1 to D 2, and move the cursor hairline to π on the C scale.
Slide C1 to the hairline and move the hairline to C 1.2.
"End switch" C10 to the hairline and move the hairline to C 1.48 to read a result of 1.12 on the D scale.
Locating the decimal point, the answer is 1.12×10^3 ohms = 1120 ohms.
Mental calculation: $2 \times 3 \times 1200 \times 0.2 = 1440$ ohms.

Exercise.

Calculate the reactance of a coil with an inductance of 390mH when connected to a supply of 16kHz. Answer: 39,200 ohms.

5. DIVISION.

5.1 The C and D scales are used for division. To divide using logarithms, the log of the divisor is subtracted from the log of the dividend, and the antilogarithm of the difference gives the quotient.

To divide using the slide rule, the scale length representing the divisor is subtracted from the scale length representing the dividend, and the quotient is read directly at C1 or C10.

5.2 When the dividend is greater than the divisor, the quotient will be greater than 1, and be read at C1. When the divisor is greater than the dividend, the quotient will be less than 1, and be read at C10. In this case, the scale reading at C10 must be divided by 10 to locate the decimal place.

Again, the method of mentally approximating the answer will also place the decimal point in the figures obtained in the answer.

Example 8. 8 ÷ 5

Set the Dividend on the D scale, using the cursor hairline.
Set the Divisor on the C scale to the hairline.
The difference between the two scale lengths will be to the left of C1.
Slide C 5 to D 8, the answer will be read at C1 on the D scale as 1.6.

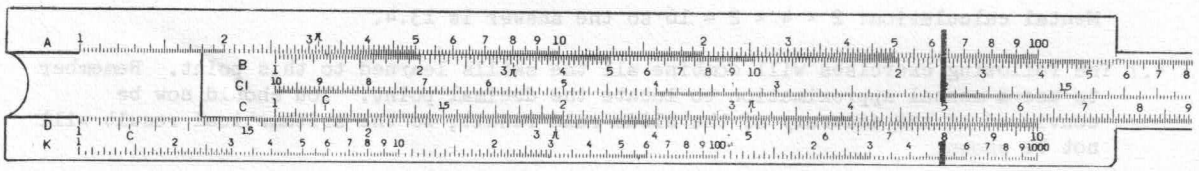

FIG. 13. EXAMPLE 8.

Note that this setting gives the result of many other division problems:

2 ÷ 1.25	3.2 ÷ 2
4 ÷ 2.5	4.8 ÷ 3
7.2 ÷ 4.5	3.94 ÷ 2.46

5.3 Work through the following exercises for practice in setting and reading:

3 ÷ 2 = 1.5	9.5 ÷ 3.74 = 2.54
4.8 ÷ 2.84 = 1.69	5.6 ÷ 1.76 = 3.18
8.4 ÷ 7.85 = 1.07	7.4 ÷ 4.7 = 1.575

Example 9. 4 ÷ 8

Set C 8 to D 4

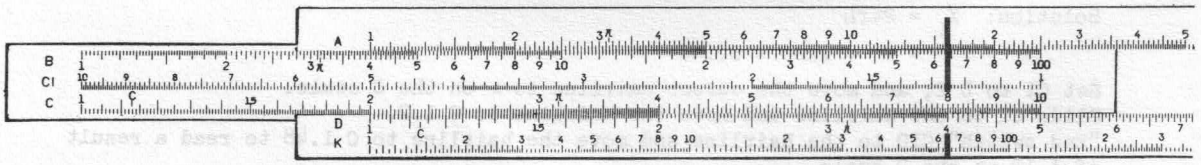

FIG. 14. EXAMPLE 9.

The answer at C1 appears on the imaginary D scale to the left of D1, and this scale would carry the numbers 0.1 to 1. We can read the answer at C10, but as this answer really lies between the values 0.1 and 1, we must read it as 0.5, not 5. Calculating mentally, 4 ÷ 8 = 0.5.

5.4 The following exercises give practice in setting and reading when C10 must be used.

Volts		ohms		amps
5	÷	8	=	0.625
2	÷	2.58	=	0.775
2.86	÷	6.15	=	0.465
1.28	÷	1.62	=	0.79
3.2	÷	5.5	=	0.582
1.93	÷	5.45	=	0.354

(Mentally 2 ÷ 2.5 = 0.8)
(" 3 ÷ 6 = 0.5)
(" 1 ÷ 1.5 = 0.66)
(" 3 ÷ 5 = 0.6)
(" 2 ÷ 5 = 0.4)

Example 10. 46.5 ÷ 2.82

This can be rewritten using powers of 10 as $4.65 \times 10 \div 2.82$ or as $\frac{4.65}{2.82} \times 10$.

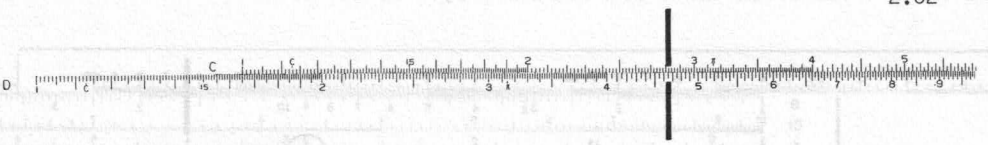

FIG. 15. EXAMPLE 10.

Set C 2.82 to D 4.65 and read the result 1.65 on the D scale.
This must now be multiplied by 10 to solve the problem, giving the answer 16.5.
Checking the location of the decimal point mentally, 46 ÷ 3 = 15 approx.

Example 11. 3.35 ÷ 43.5

Rewritten becomes $\frac{3.35}{4.35 \times 10} = \frac{3.35}{4.35} \times 10^{-1}$

Set C 4.35 to D 3.35.
Read the result 7.65 on the D scale.
This must be divided by 10, having been read at C10.
As the problem already involves a divisor of 10, the answer must be divided by $(10 \times 10) = 10^2$.
In index form 10^2 as a divisor becomes a multiplier of 10^{-2}.

Therefore the answer is $7.7 \times 10^{-2} = 0.077$.
Mental calculation would show 3 ÷ 40 = 0.075.

Example 12. 0.0765 ÷ 0.15

This problem becomes: $\frac{7.65 \times 10^{-2}}{1.5 \times 10^{-1}} = \frac{7.65}{1.5} \times 10^{-1}$

Set 1.5 on the C scale to 7.65 on the D scale.
Read the result 5.1 on the D scale, and locate the decimal point.

$$\frac{0.0765}{0.15} = 5.1 \times 10^{-1} = 0.51.$$

Example 13. 0.58 ÷ 0.0685

Rewritten becomes $\frac{5.8 \times 10^{-1}}{6.85 \times 10^{-2}} = \frac{5.8}{6.85} \times 10$

Set C 6.85 to D 5.8

Read the result 8.47 on the D scale at C10.

The division of 10 introduced by the use of C10 cancels the multiplier of 10 in the "standard form" of the equation, so the answer is 8.47

6. MULTIPLICATION AND DIVISION COMBINED.

6.1 Many problems are a combination of multiplication and division.

Simple problems involving the product of two numbers divided by a third number can be solved in one setting of the slide, by performing the division first and immediately reading off the remaining multiplication.

Example 14. $\frac{8 \times 3}{4}$

First divide by setting C 4 to D 8.
The slide is already set for multiplication,
Read off the answer on the D scale under C 3, = 6.

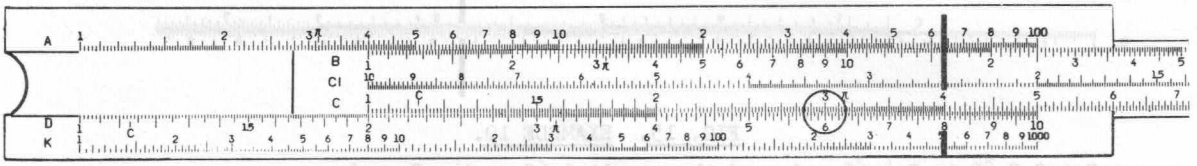

FIG. 16. EXAMPLE 14.

Note that it is unnecessary to read the intermediate result at C1 in this case.

Example 15. $\frac{3 \times 2.7}{4.5}$

Setting C 4.5 to either D 2.7 or D 3 would put C1 off the scale, (involving division by 10). The multiplication process immediately following would compensate for this. C1 to the left of D1 in multiplication involves multiplication by 10, leaving the answer on the D scale in standard form.

Mentally checking: $\frac{3 \times 3}{4.5} = 2$, Answer = 1.8.

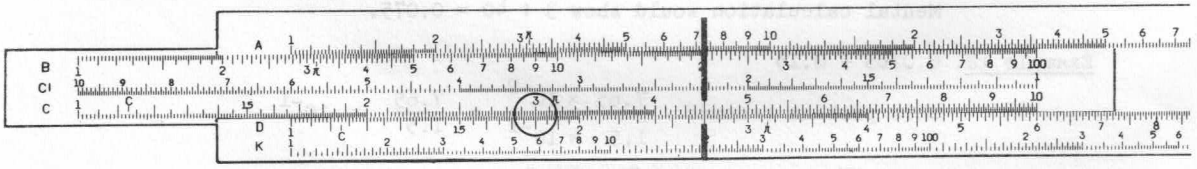

FIG. 17. EXAMPLE 15.

Example 16. Two resistors, 560Ω and 390Ω are connected in parallel. The current in the 560Ω resistor is 18.5mA. What is the current in the 390Ω resistor?

$$I_1 : I_2 :: R_2 : R_1$$

$$\text{or} \quad I_1 = \frac{I_2 R_2}{R_1}$$

$$= \frac{18.6 \times 560}{390}$$

Set C 3.9 to D 1.86. Opposite C 5.6 read 2.67 on the D scale.
Locating the decimal point by inspection,
Answer = 26.7mA.

Example 17. $\frac{2.7 \times 1.85}{5.95}$

This example cannot be solved by a single setting of the rule, and the slide must be "end-switched" to read the answer.

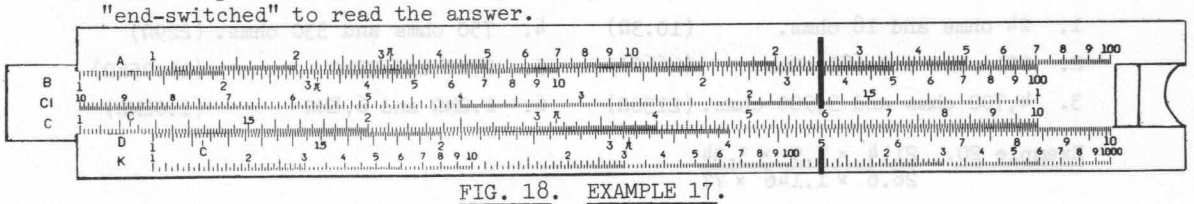

FIG. 18. EXAMPLE 17.

Perform the multiplication first, mark the result with the hairline, and slide C 5.95 to the hairline. Read the answer on the D scale at C 10.

Locate the decimal point by a mental calculation: $\frac{3 \times 2}{6} = 1$
Answer, 0.84

Example 18. $\frac{24.2 \times 7.85}{18.1}$

The solution of this problem must involve one "end-switch".
Set C 1.81 to D 2.42.
Move the hairline to C1.
Move C10 to the hairline and read the result 1.05 on the D scale at C 7.85.

Mental calculation shows $\frac{24 \times 8}{20} = 9.6$

The answer then is 10.5

Example 19. $\frac{0.535 \times 0.083}{0.254} \rightarrow 1.75$

Mental calculation, $\frac{0.5 \times 0.08}{0.2} = 0.2$

so $\frac{0.535 \times 0.083}{0.254} = 0.175$

6.2 Attempt the following exercises, paying attention to the location of the decimal point.

$$\frac{34.6 \times 0.725}{23.8} = 1.055$$

$$\frac{44 \times 3.06}{635} = 0.212$$

$$\frac{197 \times \pi}{157} = 3.94$$

$$\frac{11.75 \times 0.71}{65} = 0.1283$$

$$\frac{535 \times 1.06}{32.4} = 17.5$$

$$\frac{0.428 \times 0.81}{0.294} = 1.18$$

Solve the following problems to find the equivalent resistance of two resistors in parallel, using the formula $R_T = \frac{R_1 \times R_2}{R_1 + R_2}$

1. 24 ohms and 18 ohms. (10.3Ω)
2. 220 ohms and 680 ohms. (166Ω)
3. 4,700 ohms and 3,900 ohms. (2130Ω)
4. 750 ohms and 330 ohms. (229Ω)
5. 27kΩ and 39kΩ (15.95kΩ)
6. 1.2MΩ and 6.8MΩ (1.02MΩ)

Example 20. $\frac{21.4 \times 7.9 \times 3.34}{26.6 \times 1.146 \times 77}$

Multiply and divide in any sequence locating the decimal point by a mental calculation.

$$\frac{21.4 \times 7.9 \times 3.34}{26.6 \times 1.146 \times 77} \rightarrow \frac{20 \times 8 \times 3}{30 \times 1 \times 80} = 0.2$$

$$= 0.24$$

6.3 Proportion. Proportion problems are readily solved by slide rule.

(i) Problems of direct proportion, of the form a: b:: c: d can be solved by setting 'a' on the C scale to 'b' on the D scale, and against 'c' on the C scale reading 'd' on the D scale. Remember: Ratio a: b:: c: d
Set on scale C D C D

Example 21. 24 volts applied to a circuit cause a current of 125mA. If 39 volts were applied, what would the current be?

24 : 125 :: 39: x Answer = 203mA.

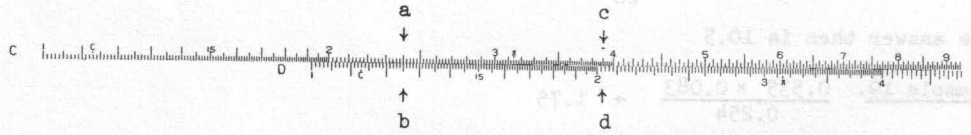

FIG. 19. READING PROPORTION PROBLEM.

(ii) Problems of inverse proportion, where one ratio is the inverse or reciprocal of the other, can be solved by first arranging the statement in the inverse ratio form. Then read the answer by setting the known values on the rule as before.

Example 22. The circuit current is 1.52 amps when the resistance is 470 ohms. If the applied e.m.f. does not change, what resistance would limit the current to 1.05 amps?
This equation is of the form $I_1 : I_2 :: R_2 : R_1$.

$$1.52 : 1.05 :: R_2 : 470$$

Set I_1 on the C scale to I_2 on the D scale.

Read R_2 on the C scale against R_1 on the D scale. **Answer** = 680 ohms.

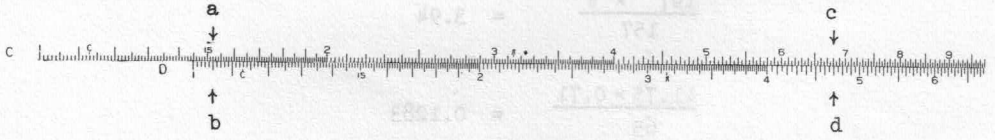

FIG. 20. READING INVERSE PROPORTION PROBLEM.

(iii) Problems of compound proportion can be set up as a mathematical statement and solved by settings on the rule. Examples and exercises of this and the preceding types of proportion problems appear in section 4 of Telecom Mathematics 1.

7. A AND B SCALES.

7.1 These identical scales are used for squaring numbers and extracting square roots.

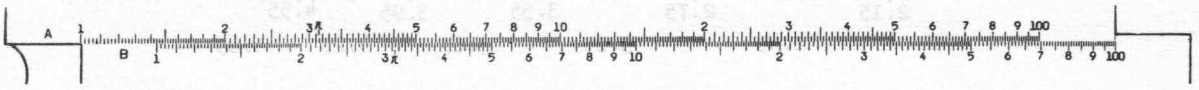

FIG. 21. A AND B SCALES OF A SLIDE RULE.

The A and B scales cover the range of numbers from 1 to 100 in the same scale length as the C or D scales cover the range 1 to 10.

Dividing a logarithm by 2 extracts the square root, so if the A or B scale is halved along its length, the halfway point would mark the square root of the number 100.

Examination of the A or B scale will show the halfway point marked 10.

The numbers from 1 to 10 are located in the left half of the A or B scale, and the numbers 10 to 100 are located in the right half of those scales.

7.2 The main subdivision of the A or B scale for the range 1 to 2 is the same length as that representing 10 to 20. Similarly the range 2 to 5 is the same length as the range 20 to 50, and again 5 to 10 the same as 50 to 100. This is reasonable, as the scales are divided logarithmically, and the mantissa of the log of 2 is the same as the mantissa of the log of 20 and so on.

The main subdivisions of both halves of the A and B scales, however, are shorter than those of the C and D scales. Consequently, the smaller subdivisions will be less detailed, hence the preference for using the C and D scales for multiplication and division.

7.3 Examine the main subdivision 1 to 2 or 10 to 20.

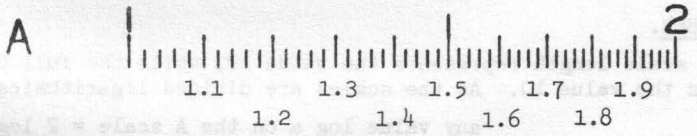

FIG. 22. A SCALE VALUES 1 TO 2.

There are ten major divisions each equal to 0.1 of the unit. The major divisions from 1 to 2 are then read as 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, and 1.9.

The major divisions from 10 to 20 would be read as 11,12,13,14,15,16,17,18 and 19.

Each major division is then subdivided into five divisions each representing 0.02 of the unit 1 to 2 or 0.2 of the unit 10 to 20.

Even numbers of three significant figures can be read directly from the scale; e.g. 1.04, 1.18, 1.36, 1.54, 1.96.

Odd numbers of three significant figures can be read by "splitting" divisions: e.g. 1.05, 1.27, 1.43, 1.75.

The corresponding values within the range 10 to 20 would be 10 times those from 1 to 2.

7.4 Examine the main subdivision 2 to 5 or 20 to 50.

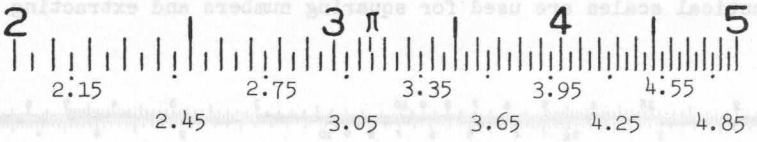

FIG. 23. A SCALE VALUES 2 TO 5.

The sections 2 to 3, 3 to 4 and 4 to 5 (similarly 20 to 30, 30 to 40 and 40 to 50) are each divided into ten major divisions each equal to 0.1 of the unit.

Each major division is then subdivided once only, so the smallest divisions have a value of 0.05 for the sections 2 to 5 and a value of 0.5 for the sections 20 to 50. Numbers which are multiples of 5 can be read to three significant figures directly from the scale.

Numbers which are not multiples of 5 have to be found by estimation, and this process has limited accuracy.

7.5 The sections 5 to 10 and 50 to 100 are also divided into ten major divisions each equal to 0.1 of the main unit, but these graduations are so close together that they are not subdivided further.

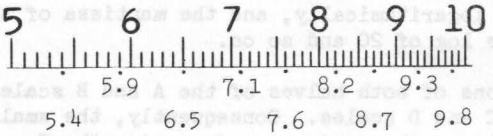

FIG. 24. A SCALE VALUES 5 TO 10.

In this section of the scale, the small divisions have a value of 0.1 for the sections 5 to 10 and a value of 1 for the sections 50 to 100.

8. SQUARING A NUMBER.

8.1 Half the A scale length represents the value 10, while the full D scale length represents the value 10. As the scales are divided logarithmically,

$$\text{any value } \underline{\log a} \text{ on the A scale} = \underline{2 \log d} \text{ on the D scale}$$

$$\text{i.e. } a = d^2$$

To read the square of any number appearing on the D scale, bring the cursor hairline to that number, and read the value of that number squared under the hairline on the A scale.

Example 23.

The most obvious example is that of reading the square of 10. Slide the hairline to D10. Read the square of 10 under the hairline at A 100.

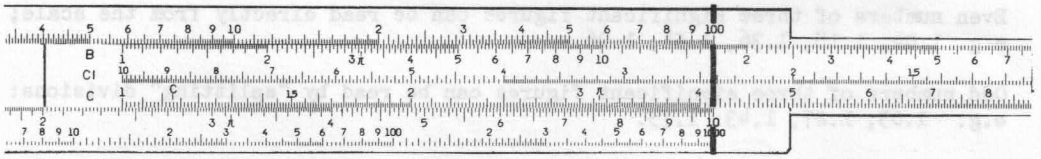

FIG. 25. READING THE SQUARE OF 10.

Note that the same result could have been found from the C and B scales.

Always be careful when reading the values of squares that a setting from the D scale is not being read on the B scale, as the slide may not be set with the index marks of each scale coinciding.

Exercise.

Read the squares of the following numbers:
(i) 2 (ii) 3 (iii) 4 (iv) 7 (v) 1.2 (vi) 4.9.

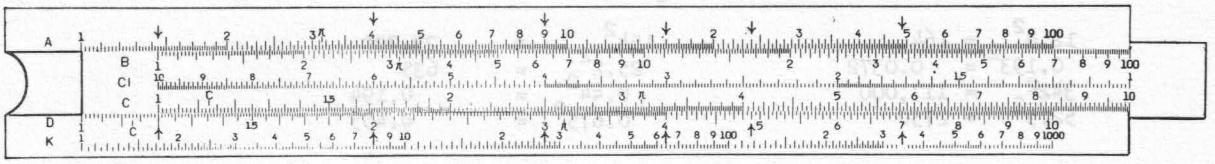

FIG. 26. READING SQUARES.

Exercises.

Read off the squares of the following numbers:

1.14^2	=	1.3		2.28^2	= 5.2
3.02^2	=	9.1		4.15^2	= 17.2
5.95^2	=	35.5		7.35^2	= 54
9.05^2	=	82		1.04^2	= 1.08

8.2 When the number to be squared is less than 1 or greater than 10, it must first be expressed as a number between 1 and 10 multiplied by a power of 10. The square of the number can then be read off the scale, and the decimal point located by multiplying the power of 10 by 2. (Note the similarity of this process to that of squaring a number by the use of logarithms).

Example 24. 15^2

Expressed in required form: $(1.5 \times 10)^2$
Move the hairline to D 1.5
Read 2.25 on the A scale under the hairline.
To locate the decimal point, multiply the power of 10 by 2.

$$10 = 10^1, \quad 10^{1 \times 2} = 10^2$$

The answer is then expressed as 2.25×10^2 or 225.

Example 25. 0.13^2

Expressed in required form: $(1.3 \times 10^{-1})^2$
Move the hairline to D 1.3
Read 1.69 on the A scale
Multiply the power of 10 by 2, giving 10^{-2}
The answer is then expressed as 1.69×10^{-2} or 0.0169.

Example 26. 42^2

Expressed in required form: $(4.2 \times 10)^2$
Move the hairline to D 4.2
Read 17.6 on the A scale.
Multiply the power of 10 by 2, giving 10^2
The answer is then expressed as 17.6×10^2 or 1760.

Example 27. 0.0715^2

Expressed in required form: $(7.15 \times 10^{-2})^2$
Move the hairline to D 7.15.
Read 51 on the A scale.
Multiply the power of 10 by 2, giving 10^{-4}.
The answer is then expressed as 51×10^{-4} or 0.0051.

Exercises.

Find the squares of the following numbers:

12.8^2	= 164	154^2	= 23,700
0.193^2	= 0.0372	25.2^2	= 635
3422^2	= 117,000	0.44^2	= 0.194
52.5^2	= 2750	0.675^2	= 0.455

8.3 Although the C and D scales are normally used for multiplication and division, the A and B scales are sometimes used, although their accuracy is limited.

Example 28. What is the power dissipated in a 2.2kΩ resistor which carries 32mA?

$P = I^2R$ where $I = 0.032$ amperes $= 3.2 \times 10^{-2}$ amps.
$R = 2{,}200$ ohms $\quad 2.2 \times 10^{3}$ amps.

First find I^2.
Move the hairline to D 3.2.
Read the square on A scale as 10.2×10^{-4}
The problem has now become
$10.2 \times 10^{-4} \times 2.2 \times 10^{3}$
Set B1 to the hairline.
Move the cursor to B 2.2.
Read the result at the hairline on the A scale as 22.5.
Solving the powers of 10 involved makes the answer $22.5 \times 10^{-1} = 2.25$ watts.

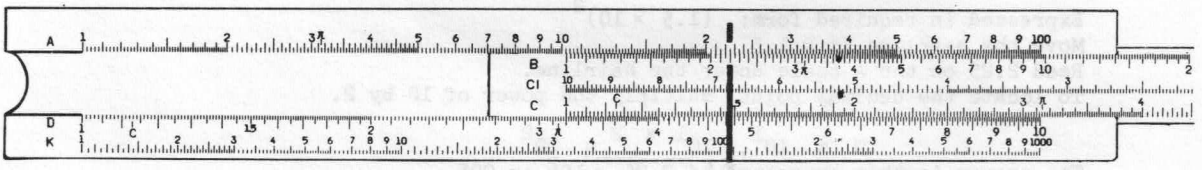

FIG. 27. READING EXAMPLE 28 ON A AND B SCALES.

Exercises. 1: What is the power dissipated in a 560 ohm resistor which carries 125mA?

Answer: 8.75 watts.

2. What is the power dissipated in a 60 ohm telephone transmitter when 47mA flows through it?

Answer: 0.132 watts.

3. Four 260 ohm resistors are connected in parallel. If the current through one resistor is 23mA, calculate the total power dissipated by the circuit.

Answer: 0.55 watts.

8.4 The gauge mark "c" engraved on the C and D scales at the point 1.128 can be used to find the area of a circle if the diameter is known.

$$\text{"c"} = \sqrt{\frac{4}{\pi}} = \frac{2}{\sqrt{\pi}} = 1.128.$$

This gauge mark "c" is derived as follows:

$$\text{Area of circle} = \pi r^2$$

$$= \pi \left(\frac{d}{2}\right)^2$$

$$= \pi \frac{d^2}{4}$$

$$= \frac{d^2}{\frac{4}{\pi}}$$

$$\text{now } c = \sqrt{\frac{4}{\pi}} \text{ so } c^2 = \frac{4}{\pi}$$

$$\text{so Area of Circle} = \frac{d^2}{c^2}$$

$$= \left(\frac{d}{c}\right)^2$$

To find the area of a circle using the mark "c", the scale length "c" is subtracted from the known diameter on the D scale, and the result read at B1 or B10 on the A scale.

Example 29. Find the area of a circle whose diameter is 2".

Set the hairline to 2 on the D scale.
Bring "c" on the C scale to the hairline.
Read the answer π on the A scale by moving the hairline to B1.

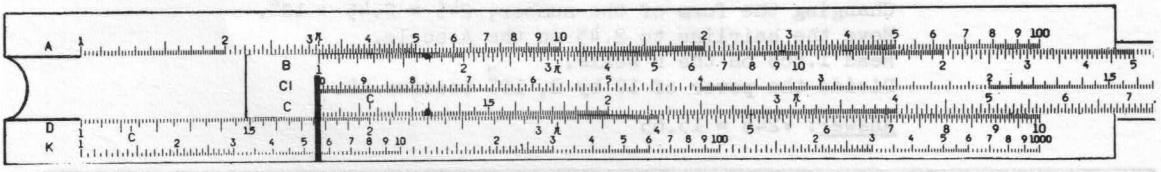

FIG. 28. READING EXAMPLE 29.

Exercises. Read the areas of the circles whose diameters are shown in the table below. (Remember to apply the rule in para. 8.2).

Diameter 3",	Area 7.1 sq. ins.	Diameter 0.214",	Area 0.036 sq. ins.
Diameter 4",	Area 12.6 sq. ins.	Diameter 0.33",	Area 0.0855 sq. ins.
Diameter 7",	Area 38.5 sq. ins.	Diameter 1.1",	Area 0.95 sq. ins.
Diameter 1.58",	Area 1.96 sq. ins.	Diameter 12",	Area 113 sq. ins.
Diameter 4.75",	Area 17.7 sq. ins.	Diameter 41.5",	Area 1350 sq. ins.

9. SQUARE ROOT OF A NUMBER.

9.1 To read the square root of any number appearing on the A scale, bring the cursor hairline to that number, and read the value of the square root of that number under the hairline on the D scale.

Example 30. Find the square root of 4.

Move the hairline to 4 on the A scale.

Read the square root of 4 under the hairline on the D scale, $\sqrt{4} = 2$.

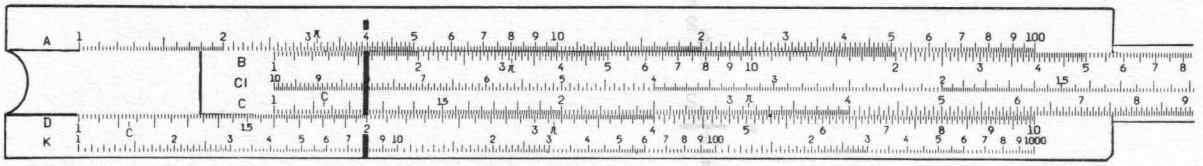

FIG. 29. READING THE SQUARE ROOT OF 4.

Example 31. Find the square root of 49.

Move the hairline to 49 on the A scale.

Read the square root of 49 under the hairline on the D scale, $\sqrt{49} = 7$.

Remember that the square root of a number on the B scale can be read directly on the C scale, but do not make the mistake of trying to read the square root from the B scale to the D scale, as the index marks may not be in alignment.

9.2 To find the square root of a number less than 1 or greater than 100, express the number in the form of a number between 1 and 100 multiplied by an even power of 10.

After the number has been expressed in this form, read the square root from the A scale to the D scale as shown in para. 9.1, and divide the power of 10 by 2 to complete the solution.

Example 32. Find the square root of 245.
Changing the form of the number, $245 = 2.45 \times 10^2$.
Move the hairline to 2.45 on the A scale.
Read 1.565 on the D scale.
Divide the power of 10 by 2, 10^2 becomes 10.

Answer: $\sqrt{245} = 15.65$

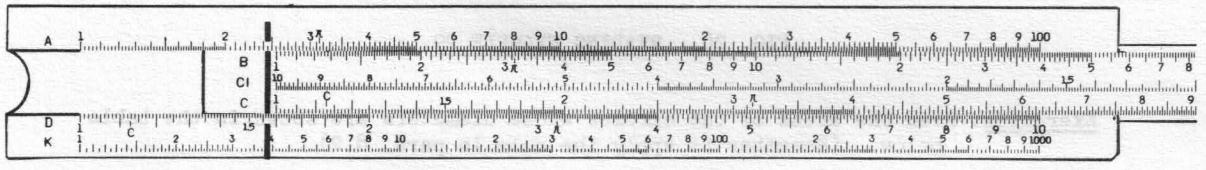

FIG. 30. EXAMPLE 32.

Example 33. Find the square root of 0.198.
Expressed as an even power of 10, $\sqrt{0.198} = \sqrt{19.8 \times 10^{-2}}$
Move the hairline to 19.8 on the A scale.
Read 4.45 on the D scale.
Divide the power of 10 by 2, 10^{-2} becomes 10^{-1}.

Answer: $\sqrt{0.198} = 4.45 \times 10^{-1} = 0.445$.

Exercise: Find the square root of the following:

0.256	=	25.6	$\times 10^{-2}$	→	5.06	$\times 10^{-1}$	=	0.506
0.468	=	46.8	$\times 10^{-2}$	→	6.84	$\times 10^{-1}$	=	0.684
0.73	=	73	$\times 10^{-2}$	→	8.54	$\times 10^{-1}$	=	0.854
0.086	=	8.6	$\times 10^{-2}$	→	2.93	$\times 10^{-1}$	=	0.293
0.028	=	2.8	$\times 10^{-2}$	→	1.672	$\times 10^{-1}$	=	0.1672
0.0016	=	16	$\times 10^{-4}$	→	4	$\times 10^{-2}$	=	0.04
0.0053	=	53	$\times 10^{-4}$	→	7.28	$\times 10^{-2}$	=	0.0728
156	=	1.56	$\times 10^{2}$	→	1.25	$\times 10$	=	12.5
394	=	3.94	$\times 10^{2}$	→	1.985	$\times 10$	=	19.85
4,625	=	46.25	$\times 10^{2}$	→	6.8	$\times 10$	=	68
6,800	=	68	$\times 10^{2}$	→	8.25	$\times 10$	=	82.5
920,000	=	92	$\times 10^{4}$	→	9.6	$\times 10^{2}$	=	960

9.3 Problems involving impedance can now be solved using the slide rule.

Example 34. A 750mH inductor is connected in series with a 4,700 ohm resistor to a source having a frequency of 1200 cycles. Find the impedance.

$$X_L = 2\pi fL = 5650 \text{ ohms.}$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$R^2 = (4.7 \times 10^3)^2 = 22 \times 10^6$$

$$X_L^2 = (5.65 \times 10^3)^2 = 32 \times 10^6$$

$$R^2 + X_L^2 = 54 \times 10^6$$

$$Z = \sqrt{54 \times 10^6} \qquad (\text{Mentally, } \sqrt{54 \times 10^6} \simeq 7 \times 10^3)$$

$$= 7.35 \times 10^3 \text{ ohms}$$

$$= 7{,}350 \text{ ohms.}$$

Example 35. A 0.22μF capacitor is connected in series with a resistor of 220 ohms, to a source having a frequency of 1800 cycles. Find the impedance.

$$X_C = \frac{1}{2\pi fC} = 402 \text{ ohms}$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$R^2 = (2.20 \times 10^2)^2 = 4.84 \times 10^4$$

$$X_C^2 = (4.02 \times 10^2)^2 = 16.2 \times 10^4$$

$$R^2 + X_C^2 = 21.04 \times 10^4$$

$$Z = \sqrt{21 \times 10^4} \qquad (\text{Mentally, } \sqrt{21 \times 10^4} \simeq 5 \times 10^2)$$

$$= 4.58 \times 10^2$$

$$= 458 \text{ ohms}$$

Exercise. A resistor of 560 ohms, a capacitor of 0.33μF and an inductor of 60mH are connected in series to a supply having a frequency of 2,400 Hz. Find the impedance of the circuit.

Answer: Z = 900 ohms.

9.4 When the values of reactance and Resistance are known, there is a short-cut method of solving the impedance of a circuit, derived in the following manner:

$$Z = \sqrt{R^2 + X^2}$$

$$= R\sqrt{\frac{R^2 + X^2}{R^2}} \qquad = X\sqrt{\frac{X^2 + R^2}{X^2}}$$

$$= R\sqrt{1 + \frac{X^2}{R^2}} \qquad = X\sqrt{1 + \frac{R^2}{X^2}}$$

$$= R\sqrt{1 + (\frac{X}{R})^2} \qquad = X\sqrt{1 + (\frac{R}{X})^2}$$

Example 36. To apply this method assume $R = 3\Omega$, $X = 4\Omega$.

(i) Set C1 to the smaller number (in this case, R) on the D scale. (The slide is now set for multiplication by R).

(ii) Set the hairline to the larger number (in this case, X) on the D scale. This gives the result of $\frac{X}{R}$ on the C scale as a number greater than 1.

(i.e. the difference between the log length of 4 and the log length of 3 is now read directly on the C scale).

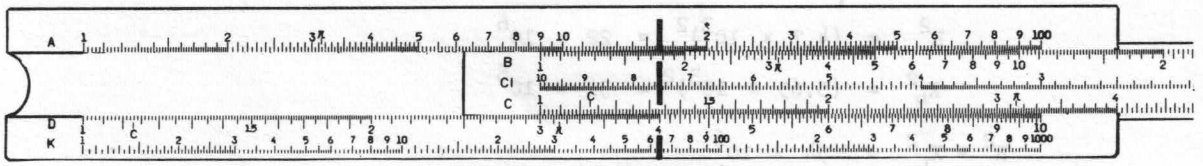

FIG. 31. EXAMPLE 36.

(iii) Read this result on the B scale under the hairline = 1.78

(This gives the result of $(\frac{X}{R})^2$)

(iv) Mentally add 1 to this result = 2.78

(Giving the expression $1 + (\frac{X}{R})^2$)

(v) Move the hairline to this new result on the B scale, 2.78

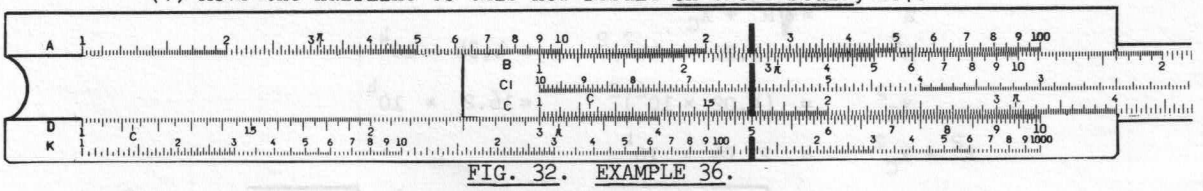

FIG. 32. EXAMPLE 36.

(vi) Read the answer on the D scale = 5

This gives the result of $\sqrt{1 + (\frac{X}{R})^2}$ multiplied by R, as set in step (i),

$$= R\sqrt{1 + (\frac{X}{R})^2}$$

Answer: Impedance = 5 ohms.

Example 37. Find the impedance of a circuit having 5 ohms Reactance and 12 ohms Resistance. (This example involves "end-switching").

Set C1 to the smaller number (5) on the D scale.

Set the hairline to the "standard form" of the larger number on the D scale. (1.2)

As the B scale is not above this setting, "end switch" the slide to place C10 at D 5.

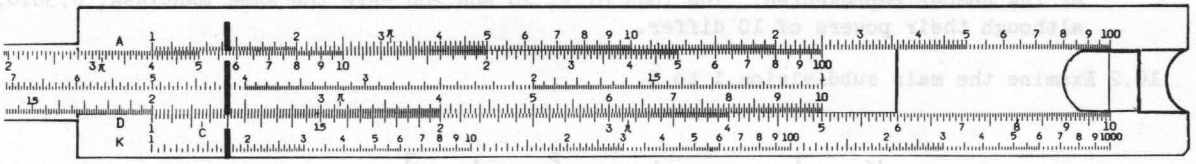

FIG. 33. EXAMPLE 37.

Now read the result on the B scale from 1.2 on the D scale = 5.75.

Mentally add 1 to this result = 6.75.

Move the hairline to this new result on the B scale.

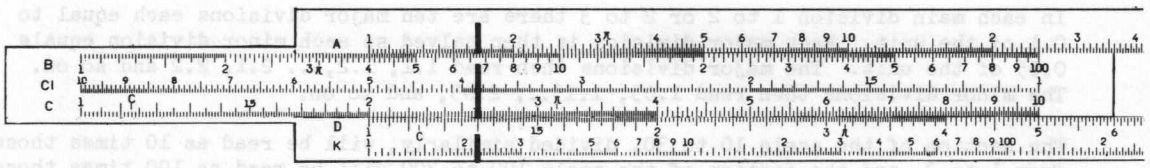

FIG. 34. EXAMPLE 37.

Read the answer on the D scale = 1.3
and locate the decimal point = 13

Answer: Impedance = 13 ohms.

Exercises. Using the values of Resistance and Reactance given below, calculate the impedance of each combination, and check your answers against the values given in the table.

Resistance in ohms	Reactance in ohms	Impedance in ohms
7	24	25
11	60	61
330	390	510
150	525	546
230	75	242
750	230	785

Note that in calculations of this type, if either factor is more than 10 times the other, the smaller factor may be neglected. This approximation will introduce an error of less than 1 per cent.

10. K SCALE.

10.1 The cube scale or K scale covers the range of numbers from 1 to 1000, in the same length as the D scale covers the range 1 to 10. The divisions of the K scale pass through the values 1 to 10 to 100 to 1000, in equal sections.

The section 1 to 10 is one third the length 1 to 1000 as $10 = 1000^{\frac{1}{3}} = \sqrt[3]{1000.}$

The section 1 to 100 is twice the length 1 to 10 as $100 = 10^{2}.$

Each section is divided identically, as the divisions are made according to the log of the number represented. The logs of 2, 20 and 200 have the same mantissa, 0.3010, although their powers of 10 differ.

10.2 Examine the main subdivision 1 to 3.

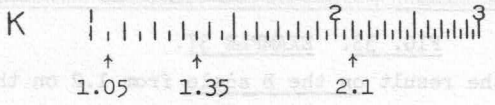

FIG. 35. K SCALE 1 TO 3.

In each main division 1 to 2 or 2 to 3 there are ten major divisions each equal to 0.1 of the unit. Each major division is then halved so each minor division equals 0.05 of the unit. The major divisions then read 1.1, 1.2,... 2.1, 2.2 and so on. The minor divisions then read 1.05, 1.15... 2.05, and so on.

The section of the scale 10 to 30, divided similarly, will be read as 10 times those from 1 to 3, and the section of the scale 100 to 300 will be read as 100 times those from 1 to 3.

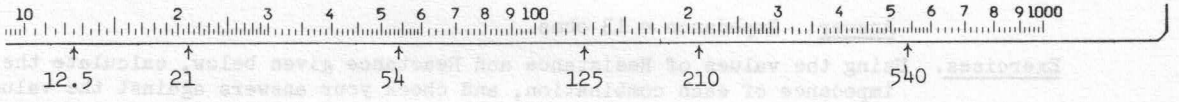

FIG. 36. READING K SCALE 10 TO 1000.

10.3 Examine the main subdivision 3 to 6.

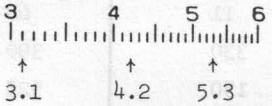

FIG. 37. K SCALE 3 TO 6.

In each main division 3 to 4, 4 to 5, 5 to 6, there are ten smaller divisions, each equal to 0.1 of the unit. These smaller divisions are read as 3.1, 3.2... 4.1, 5.1 and so on. Again, the similar section of the rule 30 to 60 will be read as 10 times those from 3 to 10 and the section 300 to 600 will be read as 100 times those from 3 to 10.

10.4 Examine the scale divisions 6 to 10.

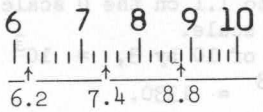

FIG. 38. K SCALE 6 TO 10.

Each main division is now divided only five times, each division equal to 0.2 of the main unit. This section of the scale must be read as 6.2, 6.4 and so on. The sections of the scale from 60 to 100 and 600 to 1000 are again similarly divided, and each division must be read as 10 times or 100 times those of 6 to 10 as before.

10.5 Cubing.

As one-third of the K scale length represents 10 while the full D scale length represents 10, any number log k on the K scale = 3 log d on the D scale,

$$\text{i.e. } k = d^3$$

(i) To read the value of the cube of any number on the D scale, move the hairline to the number on the D scale, and read the cube of that number directly on the K scale.

Example 38. Find the value of 3^3
Move the hairline to D 3
Read 27 on the K scale under the hairline.

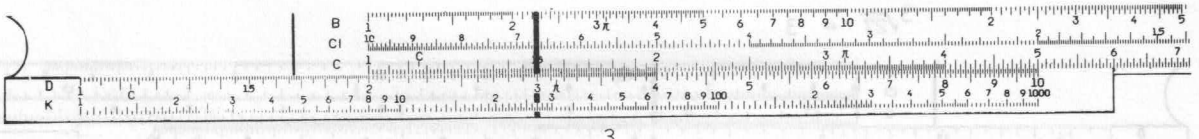

FIG. 39. READING 3^3.

Exercises: Read the following:

2^3	= 8	1.5^3	= 3.38
4^3	= 64	2.4^3	= 13.8
6^3	= 216	4.9^3	= 118
8^3	= 512	6.3^3	= 250

(ii) To read the cube of any number less than 1 or greater than 10, first express the number as a number between 1 and 10 multiplied by a power of 10, and read the cube. Locate the decimal point by multiplying the power of 10 by 3.

Example 39. 0.208^3

$0.208^3 = (2.08 \times 10^{-1})^3$

Move the hairline to 2.08 on the D scale.
Read 9 on the K scale.
Multiply the power of 10 by 3, $(10^{-1})^3 = 10^{-3}$

Answer: $9 \times 10^{-3} = 0.009$.

Example 40. 0.048^3

$0.048^3 = (4.8 \times 10^{-2})^3$

Move the hairline to 4.8 on the D scale. Read 110 on the K scale.
Multiply the power of 10 by 3, $(10^{-2})^3 = 10^{-6}$

Answer: $110 \times 10^{-6} = 0.00011$.

Example 41. 11^3

$11^3 = (1.1 \times 10)^3$

Move the hairline to 1.1 on the D scale
Read 1.33 on the K scale.
Multiply the power of 10 by 3, $= 10^3$

Answer: $1.33 \times 10^3 = 1330.$

Example 42. 252^3

$252^3 = (2.52 \times 10^2)^3$

Move the hairline to 2.52 on the D scale
Read 16 on the K scale.
Multiply the power of 10 by 3, $= 10^6$

Answer: $16 \times 10^6 = 16{,}000{,}000.$

Exercises. Find the cube of each of the following numbers:

$0.33^3 = 0.036$ $\qquad$ $15^3 = 3{,}380$
$0.212^3 = 0.0095$ $\qquad$ $30^3 = 27{,}000$
$0.067^3 = 0.0003$ $\qquad$ $525^3 = 145{,}000{,}000.$

10.6 Cube Roots.

(i) To find the cube root of any number from 1 to 1000, read directly from that number on the K scale to its cube root on the D scale.

Example 43. $\sqrt[3]{27}$

Move the hairline to 27 on K scale
Read 3 on the D scale.

$\sqrt[3]{27} = 3$

FIG. 40. READING EXAMPLE 43.

Example 44. $\sqrt[3]{145}$

Move hairline to 145 on K scale
Read 5.25 on the D scale

$\sqrt[3]{145} = 5.25$

(ii) To find the cube root of numbers less than 1 or greater than 1000, first express the number as a number between 1 and 10 multiplied by a power of 10, then arrange the power of 10 to be a multiple of 3.

This process will convert the number to one between 1 and 1000 multiplied by 10 to some power that is a multiple of 3.

Read the cube root of the number from the K scale to the D scale, and divide the power of 10 by 3 to locate the decimal point.

Example 45. Convert 0.138 and 1560 to the form required for finding the cube root.

$0.138 = 1.38 \times 10^{-1} = 138 \times 10^{-3}$

$1560 = 1.56 \times 10^3$

Exercises. Convert the following to the form suitable for finding the cube root by slide rule.

$0.0235 = 23.5 \times 10^{-3}$
$0.0076 = 7.6 \times 10^{-3}$
$0.00043 = 430 \times 10^{-6}$
$1100 = 1.1 \times 10^{3}$
$33,500 = 33.5 \times 10^{3}$
$565,100 = 565 \times 10^{3}$

Example 46. Find the cube root of 0.0235.

This number must be converted to 23.5×10^{-3}

Slide the hairline to 23.5 on the K scale

Read 2.86 on the D scale

Locating the decimal point, $\sqrt[3]{0.0235} = 2.86 \times 10^{-1}$
$= 0.286$

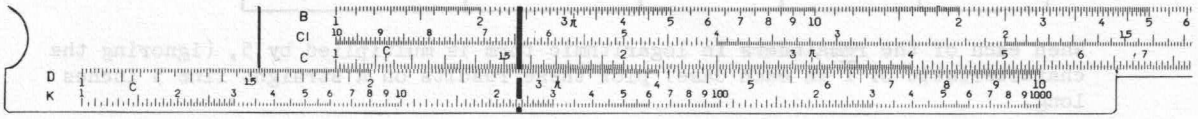

FIG. 41. READING EXAMPLE 46.

Example 47. Find the cube root of 870,000.

This number must be converted to 870×10^{3}

Slide the hairline to 870 on the K scale

Read 9.55 on the D scale,

Locating the decimal point, $\sqrt[3]{870,000} = 9.55 \times 10$
$= 95.5$

Exercises. Find the cube roots of:

0.0076	= 0.1965	1100	= 10.3
0.00043	= 0.0755	33,500	= 32.2
0.056	= 0.382	565,100	= 82.7

11. RECIPROCAL SCALE CI.

11.1 The reciprocal scale CI (pronounced C-eye) is divided in such a way that each scale division is the reciprocal of the C scale division immediately beneath it. The term "inverse" is an alternative to the term "reciprocal" so the scale is sometimes called the "Inverse C" scale, hence the designation CI.

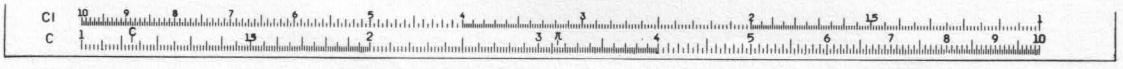

FIG. 42. C AND CI SCALES.

The divisions of the CI scale are exactly the same as those of the C scale, except they run left to right from 10 to 1, and the major divisions are designated in red. Always take care when setting a reading on this scale, as the divisions "run the other way" to those on the C and D scales.

11.2 The reciprocal of a number is that number divided into 1.
e.g. the reciprocal of 5 is $\frac{1}{5} = 0.2$.

Using logs to find the reciprocals of numbers involves three operations:

Find the log of the number
Subtract this log from zero (the log of 1)
Find the antilog of the remainder.

Apply this process to the range of numbers 1.5, 2, 3 to 9, setting the results in the form of this table:

Number	Log	Remainder	Antilog	Remainder × 5
1.5	0.176	$\bar{1}.824$	0.667	4.12
2	0.301	$\bar{1}.699$	0.5	3.495
:	:	:	:	:
9	0.954	$\bar{1}.046$	0.111	0.23

When each of the remainders in logarithmic form is multiplied by 5, (ignoring the characteristic of $\bar{1}$ in each case) plot these results on a straight line 5 inches long.

Designate each point plotted with the reciprocal the log value represented, e.g. 4.12 inches represents $\frac{1}{1.5}$.

The resultant scale should look very much like the CI scale of the rule illustrated in Fig. 1.

This exercise shows the CI scale is logarithmically divided, and graduated according to the reciprocal the division represents. Hence the designation 2 on the CI scale represents ½.

11.3 Reading Reciprocals. Immediately below the CI designation 2 is the C scale designation 5.

$$\frac{1}{2} = 0.5$$

Reciprocals can be read either by reading from the CI scale directly to the C scale, or by reading from the C scale directly to the CI scale.

Reading either way poses the problem of reading the CI scale correctly. Always remember the divisions on the CI scale "run the other way" to those on the other scales.

Adopt the following approach when reading reciprocals:

(i) Reading from C scale to CI scale, "The reciprocal of 5 is "point" 2 (0.2)."

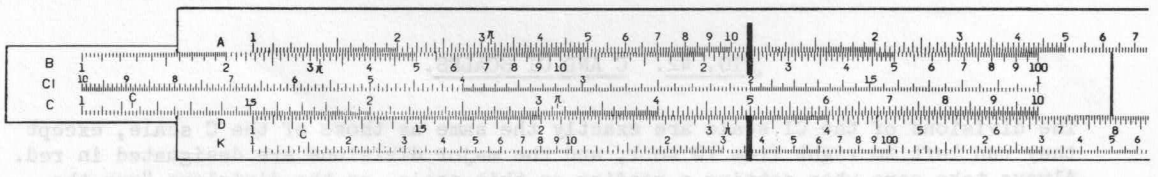

FIG. 43. READING FROM C SCALE TO CI SCALE.

(ii) Reading from CI scale to C scale,

"1 over 5 ($\frac{1}{5}$) is "point" 2 (0.2)".

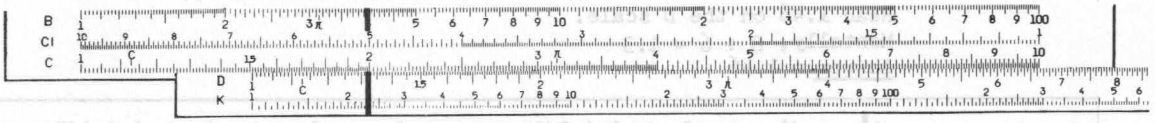

FIG. 44. READING FROM CI SCALE TO C SCALE.

Exercises. Read the reciprocals of the following numbers.

2 = 0.5	1.55 = 0.645
4 = 0.25	1.71 = 0.585
8 = 0.125	2.46 = 0.406
1.1 = 0.91	3.7 = 0.27
1.3 = 0.77	5.45 = 0.1835

11.4 Reading reciprocals of numbers less than 1 or greater than 10:-

(i) Reading Reciprocals of numbers less than 1:

convert the number to a number between 1 and 10 multiplied by a power of 10,
read the reciprocal of the number in this form,
locate the decimal point by changing the sign of the power of 10.

The reciprocal of $0.83 = \frac{1}{8.3 \times 10^{-1}} = 0.1205 \times 10$

$= 1.205.$

(ii) Reading Reciprocals of numbers greater than 10:

convert the number to a number between 1 and 10 multiplied by a power of 10,
read the reciprocal of the number in this form,
locate the decimal point by changing the sign of the power of 10.

The reciprocal of $76 = \frac{1}{7.6 \times 10} = 0.1315 \times 10^{-1}$

$= 0.01315.$

Exercises. Read the values of the following reciprocals.

$\frac{1}{255}$ ohms = 3920μmhos	$\frac{1}{0.18}$ volts = 5.55 volts
$\frac{1}{130}$ watts = 7.7mW	$\frac{1}{125}$ amps = 8mA
$\frac{1}{64}$ ohms = 15,600μmhos	$\frac{1}{0.07}$ volts = 14.3 volts

11.5 Division. Dividing by a number is the same as multiplying by the reciprocal of that number e.g. $\frac{8}{5} = 8 \times \frac{1}{5}$.

To divide using the reciprocal scale,
move CI1 or CI10 to the dividend on the D scale,
place the hairline at the divisor on the CI scale,
read the result on the D scale under the hairline,
locate the decimal point by mental calculation.

Example 48. 8.4 ÷ 5.75

Set CI1 to 8.4 on the D scale.
Place the hairline at 5.75 on CI scale (between 6 and 5)
Read 1.46 on the D scale.
Mentally, 8 ÷ 6 = 1.3
Answer = 1.46

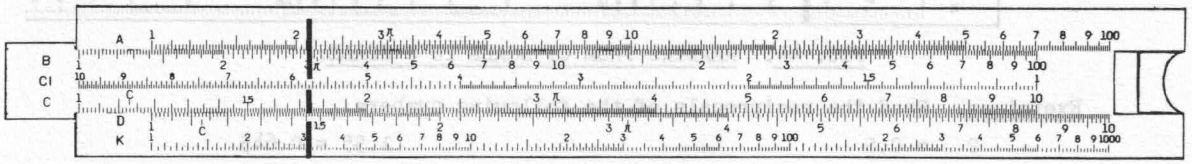

FIG. 45. EXAMPLE 48.

Exercises. Find the current in each of the following problems by the use of the CI scale.

24 volts ÷ 330 ohms = 7.27 × 10^{-2} = 72.6mA

6.3 volts ÷ 560 ohms = 1.125 × 10^{-2} = 11.25mA

48 volts ÷ 2,600 ohms = 1.845 × 10^{-2} = 18.45mA

11.6 Multiplication. Multiplying by a number is the same as dividing by the reciprocal of that number. e.g.

$$4.5 \times 2 = \frac{4.5}{\frac{1}{2}}$$

To multiply using the reciprocal scale,
locate one factor on the D scale, using the hairline;
move the other factor on the CI scale to the hairline,
read the result on the D scale at C1 or C10,
locate the decimal point by mental calculation.

Example 49. 8.3 × 4.75

Set the hairline to 8.3 on the D scale
Move CI 4.75 to the hairline
Read the result 3.94 on the D scale
Mentally 8 × 5 = 40
Answer = 39.4.

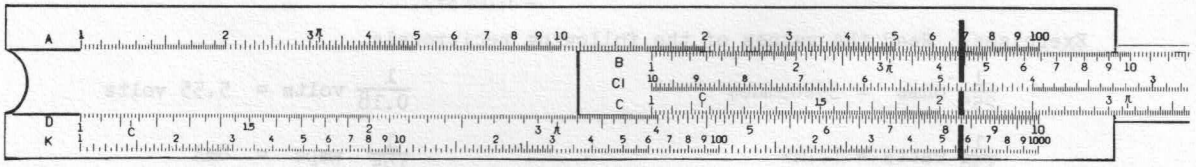

FIG. 46. EXAMPLE 49.

Exercises. Find the voltage in each of the following problems by the use of the CI scale. E = I × R

157.5mA × 2.7k = 425 volts

9mA × 5.6k = 50.4 volts

14.1mA × 33k = 465 volts

2.4 amps × 15ohms = 36 volts

1.37 amps × 62ohms = 85 volts.

11.7 Reciprocals of squares.

(i) To read the reciprocal of the square of a number between 1 and 10,

Locate the number on the CI scale,
Read the answer on the B scale.
To locate the decimal point, either multiply the reading on the B scale by 10^{-2}, or use a mental calculation.

Example 50. $\dfrac{1}{7.65^2}$

Opposite 7.65 on the CI scale, read 1.71 on the B scale.

Locating the decimal point, $\dfrac{1}{7.65^2} = 1.71 \times 10^{-2} = 0.0171$

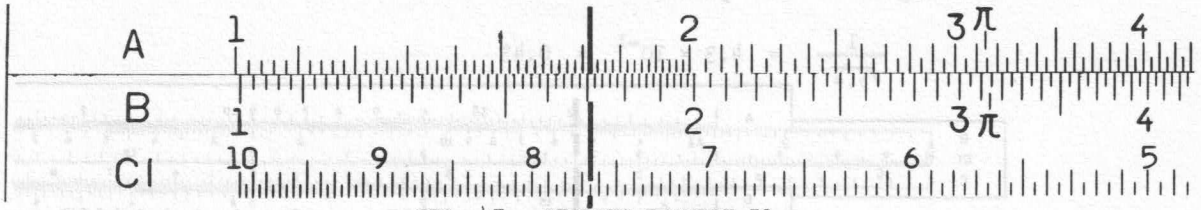

FIG. 47. READING EXAMPLE 50.

(ii) To read the reciprocal of the square of a number less than 1 or greater than 10,

Express the number as a number between 1 and 10 multiplied by a power of 10.
Locate the number on the CI scale.
Read the answer on the B scale.
To locate the decimal point:

(a) Multiply the reading on the B scale by 10^{-2}.
(b) Multiply the power of 10 in the standard form by 2 and change the sign.
(c) Solve (a) and (b) for the correct power of 10.

Alternatively, locate the decimal point by mental calculation.

Example 51. $\dfrac{1}{0.35^2}$

$$= \frac{1}{(3.5 \times 10^{-1})^2}$$

$$= 8.15 \times 10^{-2} \times 10^{2} = 8.15$$

Example 52. $\dfrac{1}{25^2}$

$$= \frac{1}{(2.5 \times 10)^2}$$

$$= 16 \times 10^{-2} \times 10^{-2}$$

$$= 16 \times 10^{-4} = 0.0016.$$

Exercise. Evaluate the following reciprocals.

$\dfrac{1}{5.2^2} = 0.037$ $\qquad$ $\dfrac{1}{0.55^2} = 3.3$

$\dfrac{1}{28^2} = 0.001275$ $\qquad$ $\dfrac{1}{0.16^2} = 39$

$\dfrac{1}{1.28^2} = 0.61$ $\qquad$ $\dfrac{1}{\pi^2} = 0.101$

11.8 Reciprocals of square roots.

(i) To read the reciprocal of the square root of a number between 1 and 100,
locate the number on the B scale,
read the corresponding number on the CI scale.
To locate the decimal point, divide the CI scale reading by 10,
or make a mental calculation.

Example 53. $\dfrac{1}{\sqrt{5.4}}$

Opposite 5.4 on the B scale, read 4.3 on the CI scale.
Locating the decimal point,

$$\frac{1}{\sqrt{5.4}} = 4.3 \times 10^{-1} = 0.43$$

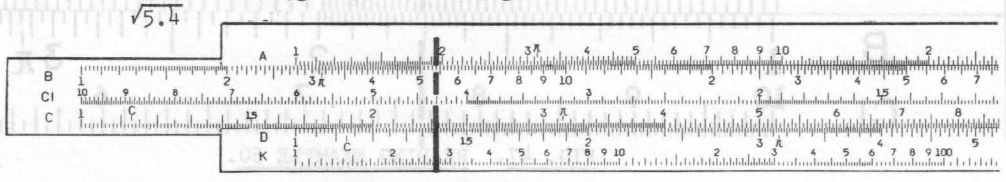

FIG. 48. READING EXAMPLE 53.

Example 54. $\dfrac{1}{\sqrt{17.6}}$

$$= 2.38 \times 10^{-1} = 0.238$$

(ii) To read the reciprocal of the square root of a number less than 1 or greater than 100, express the number as one between 1 and 100 multiplied by an even power of 10,

then locate the number on the B scale,
and read the corresponding number on the CI scale.
Locate the decimal point in the following manner;

(a) divide the CI scale reading by 10,

(b) divide the power of 10 in the reformed number by 2, and change the sign,

(c) solve (a) and (b) for the correct power of 10.

Alternatively, locate the decimal point by mental calculation.

Example 55. $\dfrac{1}{\sqrt{0.64}}$

$$= \frac{1}{\sqrt{64 \times 10^{-2}}}$$

$$= 1.25 \times 10^{-1} \times 10$$

$$= 1.25$$

Example 56. $\dfrac{1}{\sqrt{3850}}$

$$= \frac{1}{\sqrt{38.5 \times 10^{2}}}$$

$$= 1.61 \times 10^{-1} \times 10^{-1}$$

$$= 1.61 \times 10^{-2} = 0.0161.$$

Exercises. Evaluate the following reciprocals:-

$\frac{1}{\sqrt{9}} = 0.333$ $\qquad$ $\frac{1}{\sqrt{0.73}} = 1.17$

$\frac{1}{\sqrt{3}} = 0.577$ $\qquad$ $\frac{1}{\sqrt{420}} = 0.0488$

$\frac{1}{\sqrt{49}} = 0.1428$ $\qquad$ $\frac{1}{\sqrt{2350}} = 0.0206$

11.9 Inverse proportion using CI scale.

Problems of inverse proportion can be solved using the CI and D scales, just as problems of direct proportion are set and solved on the C and D scales (see para. 6.4(i)). That is, terms of the same designation are set on the same scale of the rule.

Example 57. The circuit current is 42mA when the resistance is 5600 ohms. If the applied e.m.f. does not change, what value resistor would allow the current to rise to 50mA?

$$I_1 : I_2 :: R_2 : R_1$$
$$42 : 50 :: R_2 : 5600$$

Set 42mA on CI scale against 5,600Ω on the D scale.
Move the hairline to 50mA on CI scale and read 4,700Ω on the D scale.
Place the decimal point by mental calculation.

12. L SCALE.

12.1 The L scale, generally called a "log" scale, is a scale which will provide the "mantissa" of the common logarithm of a number. (The "characteristic" is always found by inspection). The scale is divided linearly, and must be read in conjunction with the C scale on the opposite face of the slide.

12.2 The L scale is divided into ten major divisions, each of which is further subdivided twenty times. The smallest scale division therefore represents 0.005. The mantissa can then be read accurately to three significant figures.

FIG. 49. L SCALE DIVISIONS.

12.3 To find the common log of a number, e.g. 2

(i) Find the characteristic by inspection, in this case, 0.

(ii) Set C1 to the figures of the number on the D scale. (Set C1 to D2).

(iii) Turn the rule "upside-down" and read the mantissa under the hairline engraved on the "window" at the back of the slide. (Read as .301).

(iv) Combine (i) and (iii) for the answer, log of 2 = 0.301.

Exercises. Find the logs of the following numbers.

log of 3 = 0.477 $\qquad$ log of 0.284 = $\bar{1}.453$
5 = 0 699 $\qquad$ 0.047 = $\bar{2}.672$
15 = 1.176 $\qquad$ 53 = 1.724
126 = 2.10 $\qquad$ 7.5 = 0.875

12.4 To find the antilog of a number.

(i) Set the mantissa on the L scale against the hairline at the rear of the rule.

(ii) Restore the rule "face-up" and read the standard form of the number on the D scale at C1.

(iii) Locate the decimal point from the characteristic.

Example 58. Antilog 2.745

(i) Set 0.745 on the L scale to the hairline at the rear of the rule.

(ii) Invert the rule, read 5.56 at C1.

(iii) As the characteristic is 2, multiply the answer on the D scale by 10^2.

Answer: 556.

Exercises. Find the antilogs of the following numbers.

Antilog 0.185	= 1.53	Antilog 0.575	= 3.76
1.335	= 21.6	$\bar{1}$.66	= 0.457
2.455	= 285	$\bar{2}$.81	= 0.0646
0.125	= 1.334	$\bar{1}$.58	= 0.38
1.20	= 15.85	$\bar{2}$.725	= 0.053

13. S SCALE.

13.1 The S scale on the rear face of the slide is graduated in degrees and minutes, and covers the range 5°45' to 90°. Angles less than 5°45' are not shown on this rule.

Angles 5°45' to 10° are graduated in steps of 5',
Angles 10° to 20° are graduated in steps of 10',
Angles 20° to 40° are graduated in steps of 30'
Angles 40° to 70° are graduated in degrees only,
Angles 70° to 80° are graduated in 2° steps.
Angles 80° to 90° can be read only be estimation.

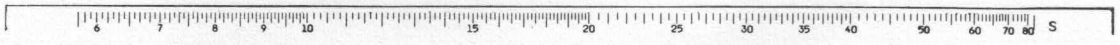

FIG. 50. S SCALE.

13.2 Sines of angles in the range of the S scale are read as decimal quantities on the C scale, as the angles from 5°45' to 90° have sine values from 0.1 to 1.

Example 59. Find the Sine of the angle 30°.

(i) Set the angle on the S scale to the hairline on the rear window of the slide.

(ii) Turn the rule face up and read the value of the sine of the angle on the C scale at D10.

Answer: Sine 30° = 0.5

Exercises. Set the following angles and read their sine value:-

Sine 6° = 0.1045	Sine 45° = 0.707
Sine 12° = 0.208	Sine 60° = 0.866
Sine 18° = 0.309	Sine 36°30' = 0.595

13.3 When the value of the sine is known, the angle can be found.

Example 60. Find the angle whose sine is 0.5.

(i) Set the sine value on the C scale to D10. (In this example set C5 to D10).

(ii) Invert the rule and read the angle at the hairline on the window.
Answer: The angle whose sine is 0.5 = 30°.

Exercises. Set the following sine values and read the angles they represent.

0.25 is the sine of $14^\circ 30'$	0.375 is the sine of 22°
0.188 is the sine of $10^\circ 50'$	0.615 is the sine of 38°
0.1435 is the sine of $8^\circ 15'$	0.875 is the sine of 61°

13.4 Cosines of angles can be read by application of the formula:- Cos α = Sine $(90^\circ - \alpha)$ where α is the angle under consideration.

The cosine values of the angles 0° to 90° range from 1 to 0, so the cosine values read on the C scale must be read as decimal quantities.

Example 61. Find the cosine of the angle 30°.

(i) Cos 30° = Sine $(90^\circ - 30^\circ)$ = sine 60°.

(ii) Set the angle 60° on the S scale to the hairline on the rear window.

(iii) Turn the rule face up and read the cosine value on the C scale at D10.
Answer: Cos 30° = 0.866.

Exercises. Find the value of the cosine of each of the following angles:-

cos 45° = 0.707	cos $75^\circ 30'$ = 0.25
cos 60° = 0.5	cos $80^\circ 40'$ = 0.162
cos 18° = 0.95	cos 59° = 0.515

13.5 When the cosine value is known, the angle can be found.

Example 62. Find the angle whose cosine is 0.695.

(i) Set the value 695 on the C scale to D10.

(ii) Invert the rule and read the angle 44°.

(iii) Subtract this angle from 90° to find the angle whose cosine is 0.695.
Answer: The angle whose cosine is 0.695 = 46°.

Exercises. Set the following cosine values and find the angles they represent.

0.530 is the cosine of 58°.	0.73 is the cosine of 43°.
0.139 is the cosine of 82°.	0.89 is the cosine of 27°.
0.39 is the cosine of 67°.	0.149 is the cosine of $81^\circ 25'$.

13.6 Problems.

1. The voltage developed by a simple A.C. generator is 3 sin ωt. What is the instantaneous e.m.f. generated when the conductor has rotated to the 55° position?

2. A 240 volts 50 cycle supply is applied to a series circuit of resistance and inductance, and the phase angle is found to be 13°. Find the P.D. across the resistance and the P.D. across the inductance.

Answer: 1, 2.46 volts. 2, E_R = 234 volts, E_L = 54 volts.

14. T SCALE.

14.1 The T scale is graduated for the angles $5^\circ\ 43'$ to 45°.

From the angle $5^\circ\ 45'$ to the angle 10° the smallest scale graduation represents 5'.

From 10° to 20° the smallest graduations represent 10'.

The smallest graduations from 20° to 45° represent 20'.

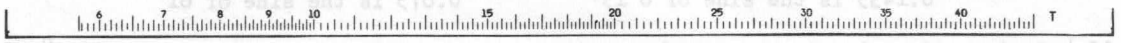

FIG. 51. T SCALE.

14.2 The tangents of the angles from $5^\circ 43'$ to 45° can be read from the C scale at D10. These tan values must be read as decimal quantities as the values for these angles range from 0.1 to 1.

Example 63. Find the value of tan 30.

(i) Set the angle 30° on the T scale to the hairline on the window at the back of the rule.

(ii) Turn the rule face up and read the value of the tangent on the C scale at D10.

Answer: $\tan 30^\circ = 0.577$.

Exercises. Read the value of the tangent of each of the following angles:-

$\tan 6^\circ = 0.105$.	$\tan 28^\circ = 0.532$.
$\tan 13^\circ 30' = 0.24$.	$\tan 34^\circ = 0.675$.
$\tan 21^\circ = 0.384$.	$\tan 41^\circ 20' = 0.88$.

14.3 The tangents of angles from 45° to $84^\circ 17'$ can also be read using the T scale. The values of these tangents range from 1 to 10 and are read on the CI scale at D10. The angle whose tangent is required is set on the T scale by observing the following rule:-

When the angle is greater than 45°, subtract the angle from 90° and set the remainder on the T scale. Then invert the rule and read the value of the tangent on the CI scale opposite D10.

Example 64. Find the value of $\tan 52^\circ$.

(i) $90^\circ - 52^\circ = 38^\circ$. Set 38° on the T scale to the window hairline.

(ii) Turn the rule face up and slide the cursor hairline to D10.

(iii) Read the tan value on the CI scale at the hairline.

Answer: $\tan 52^\circ = 1.28$.

Exercises. Find the values of the tangents of the following angles.

$\tan 46^\circ = 1.035$	$\tan 66^\circ = 2.24$
$\tan 56^\circ = 1.48$.	$\tan 70^\circ 30' = 2.82$
$\tan 60^\circ = 1.732$.	$\tan 78^\circ = 4.7$

14.4 When the value of the tangent is known, the angle can be found. The angles whose tangent values are in the range 0.1 to 10 can be read on the rule.

(i) When the tangent value known is in the range 0.1 to 1, the angle found will be in the range $5^{\circ}43'$ to 45°.

Set the known value on the C scale opposite D10, then invert the rule and read the angle at the window hairline.

Example 65. Find the angle whose tangent is 0.577.

Set 577 on the C scale against D10.
Invert the rule and read the angle 30° at the window hairline.

Exercises. Set the following tangent values and read the angles they represent.

0.136 is the tangent of $7^{\circ}45'$

0.234 is the tangent of $13^{\circ}10'$

0.37 is the tangent of $20^{\circ}20'$

0.397 is the tangent of $21^{\circ}40'$

0.735 is the tangent of $36^{\circ}20'$

0.89 is the tangent of $41^{\circ}40'$

(ii) When the tangent value known is in the range 1 to 10, the angle found will be in the range 45° to 84° 17'.

Using the cursor hairline, set the known value on the CI scale opposite D10.
Invert the rule and read the angle shown at the window hairline.
Subtract this angle from 90° to find the required angle.

Example 66. Find the angle whose tangent is 1.732.

Set 1.732 on the CI scale opposite D10.
Invert the rule and read the angle 30°.
Subtract this angle from 90° to find the required angle, 60°.

Exercises. Set the following tangent values and read the angles they represent.

1.06 is the tangent of $46^{\circ}40'$

1.32 is the tangent of $52^{\circ}50'$

2.08 is the tangent of $64^{\circ}20'$

2.7 is the tangent of $69^{\circ}40'$

4.4 is the tangent of $77^{\circ}20'$

6.7 is the tangent of $81^{\circ}30'$

15. *TEST PROBLEMS.*

1. $\frac{77.6 \times 50.2}{482 \times \pi}$ 2. $\frac{15.8 \times 46.2}{0.617 \times 6.28}$ 3. $\frac{1085 \times 60.5}{93.8 \times 4.4}$

4. $\frac{371 \times 0.24}{20.7 \times 80.5}$ 5. $\frac{0.0735 \times 47 \times 109}{8.55 \times 12.6 \times 1.96}$ 6. $\frac{0.425 \times 6.2 \times 7.6}{9.4 \times 234 \times 16.3}$

7. *A 38-tooth gear running at a speed of 270 r.p.m. drives another gear with 72 teeth. What is the speed of the driven gear?*

8. *(i)* $\sqrt{0.086}$ *(ii)* $\sqrt{0.73}$ *(iii)* $\sqrt{156}$ *(iv)* $\sqrt{4,625}$

9. *(i)* $(5.1 \times 0.34)^2$ *(ii)* $(17.1 \times 4.75)^2$ *(iii)* $(2.64 \times 19.45)^3$

10. *The resistance of a conductor is given by the formular* $R = \frac{\rho \ell}{A}$. *Find the resistance of a mile of wire having a diameter of 0.080 inches and a* ρ *of 0.634 microhms per inch cube.*

11. *A circuit having a capacitive reactance of 210 ohms has a phase angle of* 22^o. *Find the circuit impedance and resistance.*

12. *Find the instantaneous voltage (e), 3.4 milliseconds after zero voltage when* $E_{max} = 24$ *volts and* $f = 50$*Hz, given that* $e = E_{max} \sin \omega t$. *(To convert radians to degrees, multiply by 57.3).*

13. *A radio mast on top of a building is 35 feet high. The angle of elevation to the base of the mast is* 44^o, *and the angle of elevation to the top of the mast is* 48^o. *How high is the building?*

14. *In a circuit comprised of inductance and resistance, the applied alternating voltage of 180 volts has a leading phase angle of* 67^o. *Calculate the voltages across the resistance and the inductance.*

15. *Find the capacitors whose reactance at 755kHz is (i) 540Ω, (ii) 9590Ω.*

16. *If a coil with an inductance of 350mH and negligible resistance is connected across a 240 volt 50 cycle supply, how much current will pass through the coil?*

17. *A 240 volt 50 cycle supply is connected to a series circuit of 8,200 ohms resistance and 4.6 henrys inductance. Find the circuit current and the phase angle.*

18. *The input to a line 120 miles long is 61.5 milliwatts. The power delivered at the end of the line is 166 microwatts. What is the attenuation constant of the line in dB per mile?*

Answers

1. 2.57 2. 188 3. 159 4. 0.0534 5. 1.78 6. 0.000558 7. 142.5 r.p.m.

8. (i) 0.293 (ii) 0.855 (iii) 12.5 (iv) 68 9. (i) 3 (ii) 6,600 (iii) 135,000.

10. 8 ohms 11. Z = 560 ohms, R = 520 ohms. 12. 21 volts. 13. 233 feet.

14. E_R = 70.5 volts, E_L = 166 volts. 15. (i) 390pf. (ii) 22pf.

16. 2.18 amps. 17. 28.8mA, 10^o lagging. 18. 0.214db per mile.

END.